



NE 112 *Linear algebra for nanotechnology engineering*

1.6.14 Geometric series

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Outline

- In this topic, we will
 - Examine geometric series for complex ratios
 - Look at some examples





Geometric series

- Recall that for a geometric series, we had

$$\sum_{k=0}^n r^k = \frac{1 - r^{n+1}}{1 - r}$$

- If $|r| < 1$, then

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1 - r}$$





Complex ratios

- If you go back to the proof, nothing but field properties were used
 - Therefore, these formulas should be true for complex ratios

$$\begin{aligned}\sum_{k=0}^{\infty} \left(\frac{1}{2} + \frac{1}{2} j \right)^k &= \frac{1}{1 - \left(\frac{1}{2} + \frac{1}{2} j \right)} \\ &= \frac{1}{\frac{1}{2} - \frac{1}{2} j} \\ &= \frac{1}{\frac{1}{2} - \frac{1}{2} j} \cdot \frac{\frac{1}{2} + \frac{1}{2} j}{\frac{1}{2} + \frac{1}{2} j} \\ &= \frac{\frac{1}{2} + \frac{1}{2} j}{\frac{1}{2}} = 1 + j\end{aligned}$$



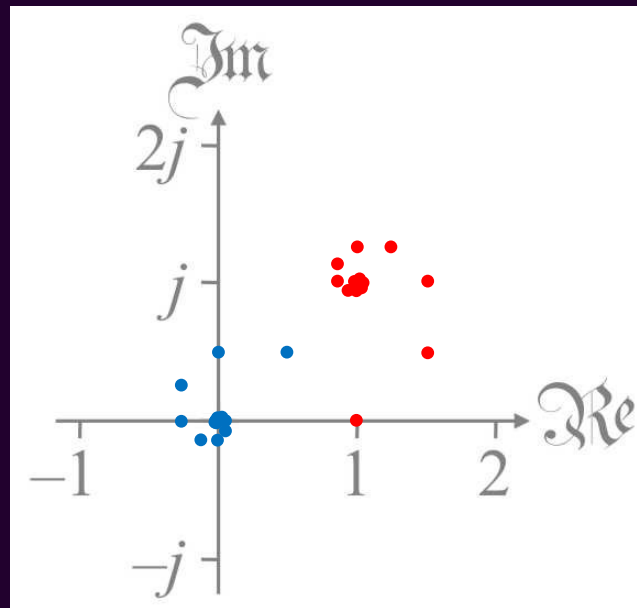


Complex ratios

- You may wonder if this is correct:

$$\sum_{k=0}^{\infty} \left(\frac{1}{2} + \frac{1}{2}j \right)^k = 1 + j$$

- You can calculate the partial sums and see this is reasonable





Complex ratios

- Similarly,

$$\sum_{k=0}^{\infty} \left(\frac{j}{\pi} \right)^k = \frac{\pi^2 + \pi j}{\pi^2 + 1} \approx 0.9080003 + 0.2890255 j$$

$$\sum_{k=0}^{\infty} \left(-\frac{1}{4} - \frac{j}{8} \right)^k = \frac{80 - 8j}{101} \approx 0.79207920 - 0.07920792 j$$





Further understanding

- Question:
 - If r is not real, will $\sum_{k=0}^{\infty} r^k$ ever be real?

- Answer: no, for

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r} = \frac{(1-r)^*}{|1-r|^2} = \frac{1-r^*}{|1-r|^2}$$





Summary

- In this topic, you now
 - Understand the formulas for geometric series hold for any field
 - Saw some examples and evidence thereof





References

- [1] https://en.wikipedia.org/wiki/Geometric_series





Acknowledgments

None so far.



Colophon

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